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Optimum profile modifications of spur gears by means of genetic algorithms

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Abstract

An original application of Genetic Algorithms (GAs) is developed in order to optimize spur gear pairs toward vibration and noise reduction. The approach takes into account the most important parameters of micro-geometric modifications, namely tip and root relief, therefore the parameter space is eight dimensional. The objective function of the GA depends on the static transmission error (STE) that is related to teeth flexibility. STE is estimated by means of a nonlinear finite element approach: either the amplitude of the STE fluctuation or its harmonic content are considered as objective functions.

The effectiveness of the approach is checked on an actual test case: GAs are able to find the optimum after a reasonable number of steps; such optimum is obtained on static basis and gives a strong vibration reduction. The reliability test proves that GAs lead to robust optima.

1. Introduction

The correlation of the noise radiated from gears with the transmission error has been clarified and described by many authors [1–3]. Most of design methods and guidelines for producing silent gears are focused on controlling and reducing sources of excitation such as the static transmission error (STE) and the manufacturing errors.

Since it is very difficult to design gears considering the actual dynamic behaviour, most of the methods for reducing gear vibration are based on static calculations.

The actual production technology allows to address the problem on three main areas: (1) macro-geometry, (2) micro-geometry and (3) surface finishing.

Macro-geometry is defined by gear parameters such as: number of teeth, diameters, pressure angle, backlash and clearance. Many authors studied the effect of the involute contact ratio ε_α (the average number of teeth in contact) on both spur and helical gear vibrations [4–6].

Micro-geometric modifications consist in an intentional removal of material from the gear teeth flanks, so that the resulting shape is no longer a perfect involute; such modifications compensate teeth deflections under load, so that the resulting transmission error is minimized for a specific torque [7].

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Surface finishing and strict manufacturing tolerance are a third way to reduce gear vibrations: manufacturing profile errors are considered as a possible source of dynamic excitation; indeed, teeth quality such as surface roughness, surfaces finishing and tolerance can play a significant role: their improvement can lead to a reduction of radiated noise.

Macro-geometric modifications involve an important and expensive change of the gear pair as well as the other members of the gear train; they are feasible only at the first steps of the design process. High-quality surface finishing and strict tolerances can lead to excessive manufacturing costs; moreover, their effect on vibrations can be disappointingly small.

Therefore, the micro-geometric optimization received great attention in the past. In the following a brief literature overview is given.

In 1940 Walker [8] considered the tooth deflection in the evaluation of the tooth load, he proposed a trapezoidal tooth load cycle from which it was possible to estimate the amount of tip relief and its extension along the tooth profile. The main contribution in this field was given by Harris [9]: he introduced the concept of STE applied to profile modifications by developing a particular diagram called ‘‘Harris map’’. According to Harris's approximation, the tooth deflection doubles its value when only one pair of teeth is in contact; this implies that, for a particular design load, the effect of the elastic deflection is exactly cancelled by a particular tip and root relief. In actual gears, such a cancellation is not complete due to the approximation of the method. In 1970 Niemann [10] developed a similar methodology for low load conditions and referred it as ‘‘short’’ relief with respect to Harris ‘‘long’’ relief. Note that neither ‘‘short’’ nor ‘‘long’’ relief can give low STE variations both at high and low loads.

The literature offers other design guidelines for profile modifications: Tavakoli and Houser [11] developed an optimization algorithm to minimize any combination of harmonics of the static transmission error, with different combinations of tip and root relief; Munro et al. [12] proposed a theoretical method for determining a set of profile modifications that gives a smooth transmission error curve, when the module of the gear is larger than 5 mm; Cai and Hayashi [13] developed an optimization technique by means of minimization of the equivalent exciting force; Matsumura et al. [14] and Rouverol [15] defined new methodologies to eradicate gears noise through profile deviations, respectively for light and high load conditions. Litvin et al. [16] recently published a paper focused on the effects of misalignement and double-crowning (surface modifications) on the vibrations of gear drives.

Interesting experiments were carried out by Kahraman and Blankenship [17] who analysed the influence of the tip relief on spur gears vibrations.

Beghini et al. [18] proposed an iterative method to reduce the peak to peak of the STE: they proposed a sequential approach, spanning couples of profile modification parameters; therefore, such a method is not an optimization technique since it is not capable to find either absolute or relative minima.

Fonseca et al. [19] used GAs to get optimal profile modifications of a spur gear pair in order to minimize the STE. An approximated formulation, based on a cantilever beam model, was used to compute the STE and the load sharing.

The analysis of the literature shows that, even though several techniques have been developed to improve the dynamic behaviour of gears, few studies were focused on global optimization approaches.

In the present work an original application of GAs has been developed to get the optimal set of profile modifications that minimizes either the peak to peak of the STE or its harmonic content. A semi-analytical FEM approach (software Calyx® [20]) is used to evaluate numerically the STE; this means that the objective function cannot be defined analytically. Furthermore, the large number of optimization parameters (four or eight according to the type of modifications) does not allow to sweep out all the domain with a reasonable computational cost. GAs solve this problem, since they find the optimum with an acceptable number of FEM calculations.

The optimal solution is checked by means of dynamic simulations; the classical one-degree-of-freedom (dof) model for spur gear pairs with time-varying mesh stiffness and backlash is considered; the equations are solved numerically and the dynamic scenario is analysed for a wide range of operating conditions. Such simulations are performed to clarify the importance of the definition of the objective function. Moreover, a reliability analysis is carried out in order to verify the robustness of optima obtained with GAs under perturbations due to manufacturing errors.

2. Dynamic model

The key point of the present study is the vibration reduction, this is pursued by optimizing the system on static basis, i.e. minimizing the excitation source due to the static transmission error fluctuation. Therefore, a dynamic model is needed to check the effectiveness of the optimization on the gear vibration; the model represented in Fig. 1 is considered to this end. Such model considers spur gears as rigid disks, coupled along the line of action through a time varying mesh stiffness $k(t)$ and a constant mesh damping c; $\theta_{g1}(t)$ is the angular position of the driver wheel (pinion), $\theta_{g2}(t)$ is the angular position of the driven wheel (gear); $T_{g1}(t)$ is the driving torque, $T_{g2}(t)$ is the breaking torque; I_{g1} and I_{g2} are the rotary inertias; d_{g1} and d_{g2} are the base diameters.

According to the literature [21] the relative dynamics of gears along the line of action can be represented by the following equation of motion:

$$m_e\ddot{x}(t) + c(\dot{x}(t)) + k(t)f_1(x(t)) + k_{\mathrm{bs}}(t)f_2(x(t)) = T_g(t), \tag{1}$$

where $(\cdot)^{\cdot} = \mathrm{d}(\cdot)/\mathrm{d}t$, m_e is the equivalent mass:

$$m_e = \left(\frac{d_{g1}^2}{4I_{g1}} + \frac{d_{g2}^2}{4I_{g2}}\right)^{-1} \tag{2}$$

$T_g(t)$ is the equivalent applied preload:

$$T_g(t) = m_e\left(\frac{d_{g1}T_{g1}(t)}{2I_{g1}} + \frac{d_{g2}T_{g2}(t)}{2I_{g2}}\right) \tag{3}$$

$T_{g2}(t) = T_{g1}(t)(d_{g2}/d_{g1})$ and $T_{g1}(t)$ are assumed to be constant.

The dynamic transmission error $x(t)$ along the line of action is defined as

$$x(t) = \frac{d_{g1}}{2}\theta_{g1}(t) - \frac{d_{g2}}{2}\theta_{g2}(t) \tag{4}$$

$k_{\mathrm{bs}}(t)$ is the back side contact stiffness; it is to note that such kind of contact is rarely observed in the case of high speed and high loads, it is considered here for completeness.

Smoothing backlash functions are considered in order to simulate clearances:

$$\begin{aligned} f_1(t) &= \tfrac{1}{2}[(x(t)-b)\{1+\tanh[\lambda(x(t)-b)]\}], \\ f_2(t) &= \tfrac{1}{2}[(x(t)+b)\{1+\tanh[-\lambda(x(t)+b)]\}], \end{aligned} \tag{5}$$

where $2b$ is the backlash along the line of action and λ is the shape parameter ($\lambda = 10^8$), the accuracy of smoothing technique has been proven in Fig. 6 of Ref. [22].

The gear pair mesh stiffness along the line of action is given by

$$k(t) = \frac{2T_{g1}}{d_{g1}\mathrm{STE}(t)} = \frac{4T_{g1}}{d_{g1}^2\delta(t)}, \tag{6}$$

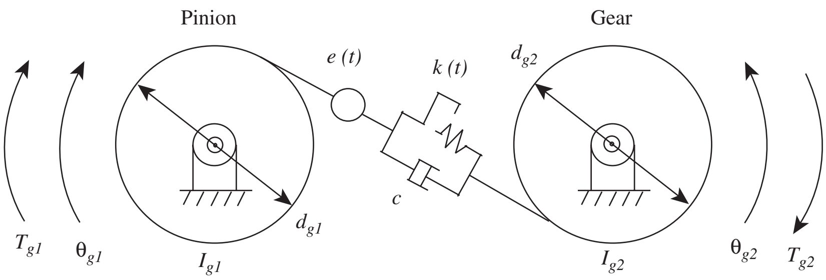

Fig. 1. Dynamic model of a spur gear pair.

where

$$\delta(t) = \theta_1(t) - \frac{d_{g2}}{d_{g1}}\theta_2(t) \quad \text{and} \quad \mathrm{STE}(t) = \frac{d_{g1}\delta(t)}{2} \tag{7}$$

$\delta(t)$ is the difference between the nominal position of the wheel 1 (pinion) given by the exact kinematics and the actual position influenced by the teeth flexibility; STE(t) is the static transmission error along the line of action, it depends on time because, during meshing, the reciprocal position of wheels, the contact point and the number of teeth in contact can change.

Parker et al. [23] described a methodology to compute $\theta_1(t)$ and $\theta_2(t)$, referred as the rotational dofs of the pinion and the gear, for small "rigid-body" motions [24]; this approach is followed here for static analyses.

Since no manufacturing errors are included, the mesh stiffness is periodic within a mesh cycle; therefore, it is expanded in terms of Fourier series:

$$k(t) = k_0 + \sum_{j=1}^{N} k_j \cos(j\omega_m t - \varphi_j), \tag{8}$$

where ω_m is the mesh circular frequency, amplitudes k_j and phases φ_j are obtained using the discrete Fourier transform (DFT); the number of samples n is related to the number of harmonics $N = (n-1)/2$; in the following, $n = 15$ is considered to ensure enough accuracy in the expansion.

Similarly we have

$$k_{\mathrm{bs}}(t) = k_0 + \sum_{j=1}^{N} k_j \cos\left(-\left(j\omega_m t - \varphi_j + \frac{s_{\mathrm{ts},1}}{d_{g1}}\right)\right), \tag{9}$$

where $s_{\mathrm{ts},1}$ is the thickness of the pinion tooth space at the pitch operating diameter, see Eq. (3.2.32) of Ref. [21] for details.

A dimensionless form of Eq. (1) is obtained by letting:

$$\omega_n = \sqrt{\frac{k_0}{m_e}}; \quad \zeta = \frac{c}{2m_e\omega_n}; \quad \tau = \omega_n t; \quad \bar{T}_g = \frac{T_g}{bm_e\omega_n^2}; \quad \bar{x} = \frac{x}{b} \tag{10}$$

and

$$\bar{k}_j = \frac{k_j}{m_e\omega_n^2}; \quad \bar{k}(\tau) = 1 + \sum_{j=1}^{N} \bar{k}_j \cos\left(j\frac{\omega_m}{\omega_n}\tau - \varphi_j\right),$$

$$\bar{k}_{\mathrm{bs}}(\tau) = 1 + \sum_{j=1}^{N} \bar{k}_j \cos\left(-j\frac{\omega_m}{\omega_n}\tau + \varphi_j - \frac{s_{\mathrm{ts},1}}{d_{g1}}\right). \tag{11}$$

3. Profile modifications

Fig. 2(a) shows standard profile modifications on a spur gear tooth, which consist in a removal of material from the tip (tip relief) or the root (root relief), according to different manufacturing parameters. The "start roll angle at tip" α_{ts} and the "magnitude at tip" mag$_t$ specify the point on the profile at which the relief starts and the amount of material removed at the tip radius; α_{rs}, mag$_r$ and α_{re} have similar meaning, where the current roll angle α is given by $\alpha = \sqrt{(d/d_{g1})^2 - 1}$, see Fig. 2. Typical manufacturing gear processes, such as grinding, allow to control whether the variation of the removed material is linear or parabolic with respect to the roll angle. Since the removal of material is measured along the direction normal to the profile, usual representations of the reliefs are given as deviation from the theoretical involute profile: Figs. 2(c,d) show examples of linear and parabolic modifications.

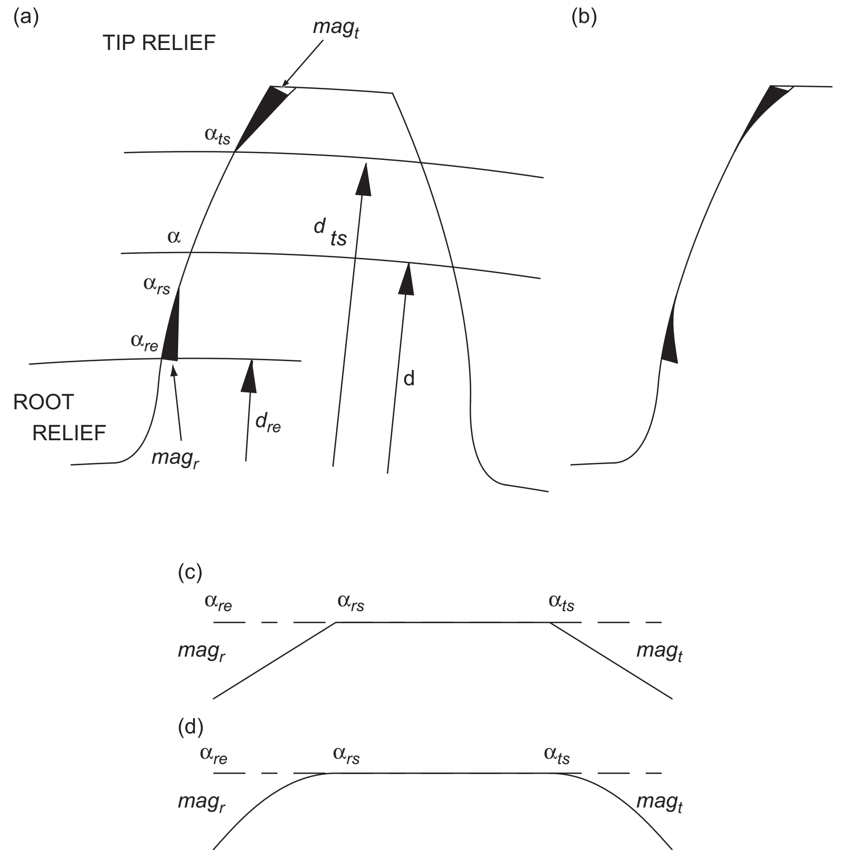

Fig. 2. Representation of profile modification parameters. Tooth shape: (a) linear modifications; (b) parabolic modifications. K-chart: (c) linear modifications; (d) parabolic modifications.

Table 1
Parameter ranges for the genetic optimization

Parameter	Start	End
α_{ts}	Roll angle at operating pitch diameter	Roll angle at tip diameter
mag_t	0	40 μm
α_{rs}	Roll angle at operating pitch diameter	Roll angle at initial point of contact diameter
mag_r	0	40 μm

In the present work the following assumptions are made: (1) the type of profile modification (linear or parabolic) is chosen before the optimization process and remains unchanged; (2) the "end roll angle at root" α_{re} is the roll angle corresponding to the diameter of the initial point of contact along the tooth profile; (3) 2D plain strain FEM analyses are carried out; therefore, no crowning effects are taken into account.

4. Formulation of the genetic algorithm

The present genetic algorithm is based on a binary encoding of eight parameters, which identify the set of profile modifications on both pinion and gear profiles. Ranges spanned by each parameter are reported in Table 1. Such intervals are sampled using 11 bits (2048 samples) for roll angles and 6 bit (64 samples) for

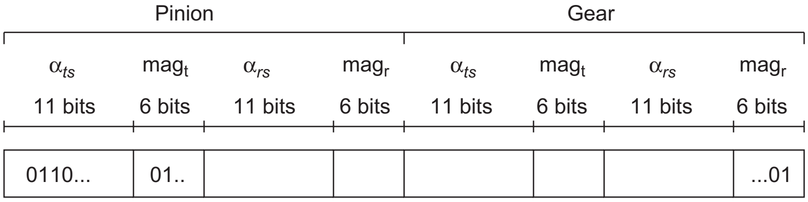

Fig. 3. Encoding of the profile modifications in a binary string S.

magnitudes; this sampling allows to obtain a discretization step less than 1 μm for the magnitude and less than 2 μm for the roll angle diameter d.

The total number of bits needed to encode all parameters is $nbits = 68$; Fig. 3 shows a graphical representation of the string. The position of the parameters within the string is set in order to maintain pairs of correlated variables at the maximum distance from each other; this increases the variation within the parameter space. For example, the effect of the gear magnitude at tip mag$_t$ on the STE, is strongly correlated to the pinion magnitude at root mag$_r$, because, during meshing, the pinion root will be in contact with the gear tip.

Once an initial population of strings is randomly built up for the first trial case, the numerical evaluation of the STE is performed.

Two different binary codes are now considered: the standard binary code and the Grey code. A general string S, consisting of n_{var} substrings s_i having z_i bits, can be decoded into an array X of n_{var} real parameters x_i.

$$S = s_1 s_2 \dots s_{n_{\text{var}}} = a_1 a_2 \dots a_m \quad \text{with} \quad a_j \in \{0,1\}; \quad m = \sum_{k=1}^{n_{\text{var}}} z_k$$
$$X = [x_1,\ x_2, \dots, x_{n_{\text{var}}}]^{\mathrm{T}}, \tag{12}$$

The decoding of a substring s_i can be carried out through the standard binary encoding

$$x_i = x_i^{(1)} + \frac{x_i^{(2)} - x_i^{(1)}}{2^{z_i}} \sum_{j=1}^{z_i} a_{i^*_{i,j}} 2^{j-1} \quad \text{with} \quad x_i \in [x_i^{(1)}, x_i^{(2)}]$$
$$i^*_{i,j} = \left(\sum_{k=1}^{i-1} z_i \right) + z_i - j + 1, \tag{13}$$

where $x_i^{(1)}$ and $x_i^{(2)}$ define the domain of existence of the i-th variable in a non-constrained problem.

Similarly, for the Grey encoding we have:

$$x_i = x_i^{(1)} + \frac{x_i^{(2)} - x_i^{(1)}}{2^{z_i}} \sum_{j=1}^{z_i} \left(\bigoplus_{k=1}^{z_i - j - 1} a_{i^*_{i,k}} \right) 2^{j-1} \quad \text{with} \quad x_i \in [x_i^{(1)}, x_i^{(2)}]$$
$$i^*_{i,k} = \left(\sum_{k=1}^{i-1} z_i \right) + k, \tag{14}$$

where $\bigoplus$ indicates addition modulo two.

The Grey encoding is a technique for representing integers using the base two; its special feature is that only one bit changes between the representations of two subsequent integers. According to Bäck et al. [25], this property increases the performance of GAs during transformations.

The present GA improves solutions by means of a certain number of iterations n_{iter} on a population of $n_{\text{pop}} = 50$ strings. The first population is randomly generated; each string is decoded and associated to the corresponding value of the objective function. This value is called "Fitness", because it is the guideline parameter, which allows to find better solutions, in the same way as fitness generates the best individuals of a biological population.

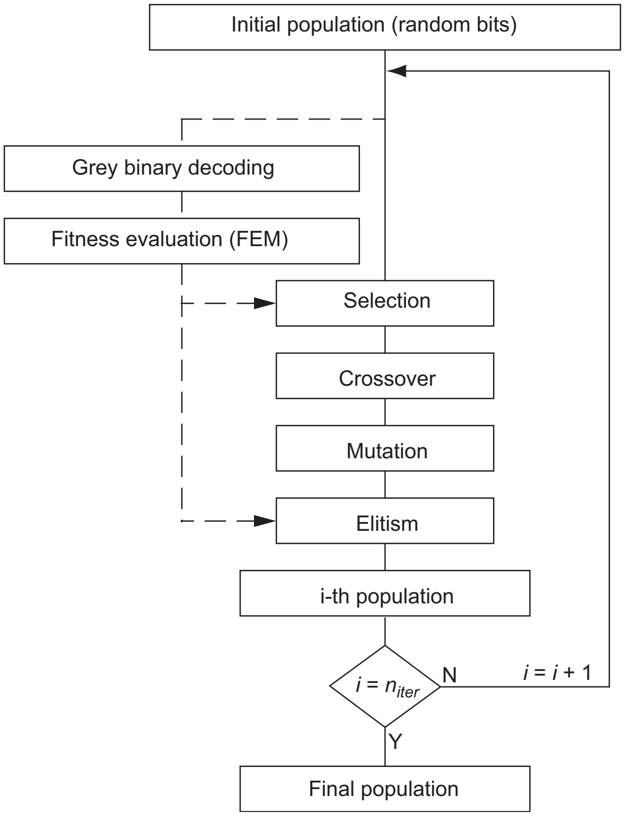

Fig. 4. Flow chart of the Genetic Algorithm.

Each iteration is made of three steps: Selection, Crossover and Mutation, see Fig. 4.

The Selection produces a new population by extracting n_{pop} strings from the previous one. Extractions are carried out as in a cheating roulette in which strings having high fitness have higher probability to be extracted. Here, the probability of extraction of a certain string is defined as the ratio between the string fitness and the sum of the fitness of all strings within the prior population. A variation of the basic selection approach, called "stochastic reminder selection without replacement", has been used here because it is less sensitive to stochastic variations [26].

Once a new population is generated, strings are randomly grouped by two; for each pair a cutting point is randomly selected; the cutting point can assume a value from 1 to $m-1$. The first part of the father string and the second part of the mother string are combined to obtain the first child string; a second child string is created with the remaining parts. This process is called Crossover. A peculiarity of the presented approach is that all strings are used and none is repeated. A crossover probability index p_c can be considered in order to control the number of crossing string pairs, i.e. the velocity of variation of the population. Indeed, the crossover guarantees variability within high fitness regions.

In order to avoid that some regions are not spanned by the GA, a final step, the Mutation, completes the algorithm: each bit is changed according to a mutation probability p_m.

At the end of each iteration, the best string of the old population replaces the worst string of the new one. This expedient, called elitism, gives advantages in the case of a large population, by preserving good solutions from unwanted elimination.

During iterations, the fitness f tends progressively towards a specific value; at the same time, the difference between the average $\bar{f}$ and the maximum $f_{\max}$ of the fitness, within the population, drops down. All strings tend asymptotically to the same value, thus drastically reducing the efficiency of the algorithm in selecting the best string. Furthermore, the fitness must be positive.

The fitness scaling solves the aforementioned problems by means of a linear transformation:

$$f' = af + b \tag{15}$$

coefficients a and b are chosen in order to match the following requirements: the average of the fitness must remain unaltered and the extraction probability of the string must have the maximum fitness equal to a certain value c_{mult}:

$$a = \frac{\bar{f}(1 - c_{\mathrm{mult}})}{\bar{f} - f_{\max}}; \quad b = \frac{\bar{f}(\bar{f}c_{\mathrm{mult}} - f_{\max})}{\bar{f} - f_{\max}}. \tag{16}$$

If the minimum of the scaled fitness $f'_{\min}$ is negative, coefficients a and b are recalculated imposing that the average fitness remains unaltered and $f'_{\min} = 0$:

$$a = \frac{\bar{f}}{\bar{f} - f_{\min}}; \quad b = \frac{\bar{f} f_{\min}}{\bar{f} - f_{\min}}. \tag{17}$$

The limit condition between the previous two cases is evaluated through the following inequality:

$$f_{\min} \geqslant \frac{f_{\max} - \bar{f}c_{\mathrm{mult}}}{1 - c_{\mathrm{mult}}}. \tag{18}$$

Since no convergence criteria can be defined for GAs, the evolution of the population during genetic iterations should be checked. To this purpose, De Jong [27] defined the online performance r_{ON} and offline performance r_{OFF} as follows:

$$r_{\mathrm{ON}}(h) = \frac{1}{h}\sum_{k=1}^{j} f(X_h) \quad \text{with} \quad h = 1, \ldots n_{\mathrm{iter}} n_{\mathrm{pop}}, \tag{19}$$

$$r_{\mathrm{OFF}}(h) = \frac{1}{h}\sum_{k=1}^{h} f_k^* \quad \text{where} \quad f_k^* = \max\{f(X_i) : i = 1, \ldots, k\}, \tag{20}$$

$h = 1, \ldots n_{\mathrm{iter}} n_{\mathrm{pop}}$ is an index that counts all strings, without regard to populations.

The online performance represents the mean of the fitness within iterations, the offline performance is the average fitness of the maxima found at the previous iterations; the latter one is the most suitable parameter to control the optimization in maximum–minimum problems.

In the present work, both offline and online performances have been checked as a posteriori convergence criterion.

5. Gear profile optimization

In the present section the GA is applied to optimize the spur gear pair described in Table 2. Note that in the original case study, profile modifications are not present; in the following it will be referred as case A. In this case, the natural frequency of the system is $\omega_n = 3.156 \times 10^4\,\mathrm{rad/s}$.

Three different optimizations are performed using the peak to peak value of the STE as objective function and different values of p_c, p_m and c_{mult} (cases B, C and D). A fourth optimization (case E) is carried out using the average of the first seven harmonic components of the STE as objective function. For each case, the parameters of the GA have been set according to Table 3.

Parameters p_c, p_m and c_{mult}, for cases B and C, have been taken from literature [27]. For the case D, $p_c = 1$ means that all strings are involved during crossover, while p_m has been optimized after some trial simulations. Simulations with more then 800 iterations show that $n_{\mathrm{iter}} = 100$ guarantees sufficient convergence to the solution.

Table 4 shows the reduction of the STE peak to peak for cases B, C and D with respect to case A: the algorithm is more efficient using Grey encoding; parabolic (quadratic) modifications are less effective than the linear one, even though they are much more expensive to manufacture.

Table 2
Geometrical data for the case study (courtesy of CNH Case New Holland)

Data	Pinion	Gear
Number of teeth	28	43
Module (mm)	3	3
Pressure angle (deg)	20	20
Base radius (mm)	39.467	60.610
Theoretical pitch radius (mm)	42	64.5
Thickness on theoretical pitch circle (mm)	6.1151	6.7128
Addendum modification (mm)	1.927	2.748
Face width (mm)	20	20
Hob tip radius (mm)	0.9	0.9
Outer diameter (mm)	93.1	139.7
Root diameter (mm)	79.1	126.2
Inner diameter (mm)	40	40
Mass (kg)	0.71681	1.9823
Inertia (kg m^2)	0.0008076	0.0047762
Young's modulus (MPa)	206000	206000
Poisson's coefficient	0.3	0.3
Centre distance (mm)	111	
Backlash (mm)	0.3461	
Backlash ($2b$) on the line of action (mm)	0.312	
Backside stiffness phase (rad)	1.594232	
Transmission ratio	0.6511	
Contact ratio	1.28565	
Torque (T_{g1}) (N mm)	470000	
Damping coefficient (ζ)	0.01	

Table 3
Parameters of the genetic optimization

	Case B	Case C	Case D	Case E
Number of strings in the population n_{pop}	50			
Crossover probability p_c	0.6	0.6	1	1
Mutation rate p_m	0.033	0.033	0.04	0.04
Multiplier for the fitness scaling c_{mult}	1.5			
Number of iterations n_{iter}	100			
Type of modification	Linear	Parabolic	Linear	Linear
Binary code	Standard	Standard	Grey	Grey
Objective function	Peak to peak of STE			Average of harmonics of the STE

Table 4
Peak to peak of the static transmission error

	Case A	Case B	Case C	Case D
STE peak to peak (μm)	11.6457	1.73964	2.60228	1.6323
Reduction	–	85.06%	77.65%	85.98%

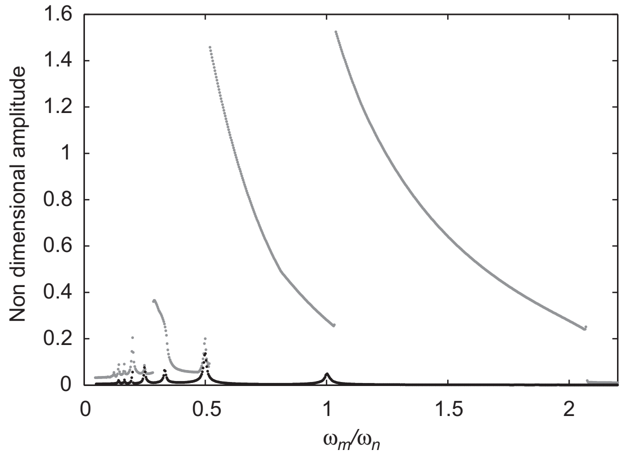

Fig. 5. Amplitude frequency diagram: case A (grey line); case B (black line). Backward simulation only.

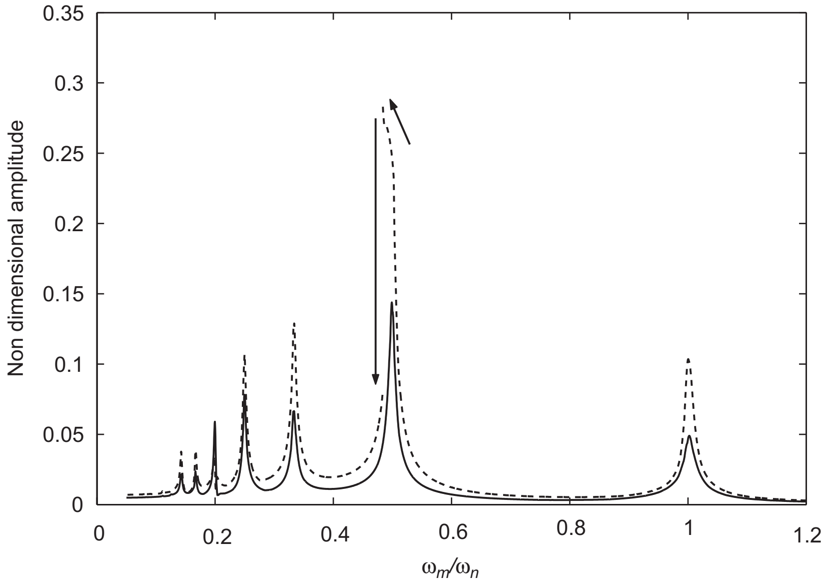

Fig. 6. Amplitude frequency diagram: case B, linear modifications (solid line); case C, parabolic modifications (dashed line). Backward simulation only.

A dynamic analysis shows that the scenario changes dramatically due to the optimization. In absence of profile modifications, see Fig. 5 grey line, gears experience the following dynamics: (1) a period doubling (PD) bifurcation ($\omega_m/\omega_n = 2.07$) due to parametric instability, which leads to a huge amplification of oscillation with contact loosing; (2) fundamental resonance ($\omega_m/\omega_n \approx 1$) having a strongly softening character and leading to large amplification of oscillation (the nonlinearity means that a contact loosing took place); (3) peaks at low excitation frequency due to superharmonic resonances related to the presence of higher harmonics of $k(t)$. After optimizing, in the case B, see Fig. 5 black line, the PD bifurcation completely disappears and the fundamental resonance is strongly reduced. Resonances at low frequencies are reduced as well; in this case the improvement is smaller than in the case of the fundamental resonance.

Using parabolic profile modifications, case C, the dynamic performance is worst than the linear modifications, Fig. 6; such behaviour is not surprising in the view of the peak to peak of the STE, Table 4.

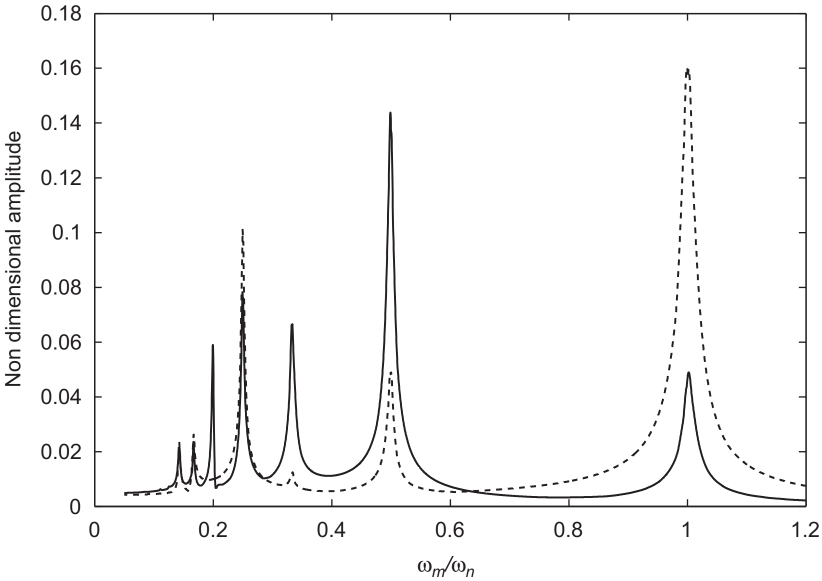

Fig. 7. Amplitude frequency diagram: case B (solid line); case D (dashed line). Backward simulation only.

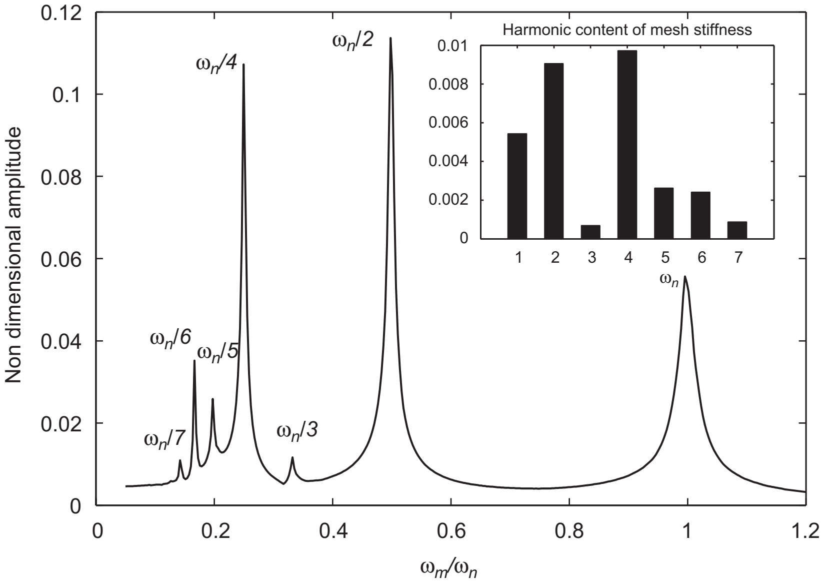

Fig. 8. Amplitude frequency diagram: case E. Backward simulation only.

In particular, even if the parametric instability disappears, there is contact loosing at $\omega_m/\omega_n = 0.5$ and all the superharmonic resonances lead to higher amplitudes.

Using Grey encoding and linear profile modifications leads to satisfactory results, which are comparable to the case B on the average. Indeed, there is an improvement at low frequencies, Fig. 7, even though the principal resonance amplitude is a bit larger; however, no contact loosing is observed as well as parametric instability.

Using the harmonic content of $k(t)$ as objective function leads to the best result on the average. Indeed, the parametric instability disappears, similarly to the other cases, and the amplitude of oscillation is smaller than

Table 5
Optimal profile modifications according to optimizations: cases D and E

	Case D		Case E	
	Pinion	Gear	Pinion	Gear
Tip relief				
α_{ts} (deg)	31.504	30.036	31.941	29.498
mag_t (mm)	0.020317	0.023492	0.003809	0.014603
Root relief				
α_{rs} (deg)	19.943	24.448	24.698	24.337
α_{re} (deg)	14.433	20.576	14.433	20.576
mag_r (mm)	0.002539	0.006349	0.021587	0.025397

$0.12\,b$ over the whole frequency range (Fig. 8); there is a direct proportionality between the harmonic content of the mesh stiffness and the amplitude of resonance peaks, which reflects the linear behaviour of the gear pair with profile modifications.

It must be noticed that the optimized mesh stiffness is essentially an even function (small box in Fig. 8): all simulations (cases B–E) lead to high even frequencies in the spectrum of the mesh stiffness for the optimized solution. Since the mesh stiffness function origin is located at the pitch point, an even shape corresponds to a symmetric behaviour between the tooth approach and recession. In this sense, the genetic optimization technique yields to similar results for uncorrelated configurations such as "approach" and "recession".

Table 5 summarizes the profile modification parameters for the two best cases: the two sets of parameters are apparently quite different from each other; however, they have a sort of symmetry that can be physically explained considering that the reduction of tip relief on the pinion is compensated by an increasing of root relief on the gear. In other words, the set D concentrates modifications on the tip and set E concentrates modifications on the root.

Profile modifications, proposed in Table 5, are acceptable also from a technological point of view; indeed, that gear manufacturing tolerances, such as "K" charts, are usually of the order of some micrometre.

6. Reliability analysis

The GA methodology defines an optimal set of theoretical profile modifications; however, in actual gears, manufacturing errors cannot be avoided and represent a perturbation of the optimum; an important and final step of a reliable optimization process is to evaluate how much the objective function is sensitive to perturbations, i.e. the robustness of the process.

GAs are not suitable to find strongly localized minima, because they perform a search preferably in high fitness area; if the minimum is strongly localized, it is not surrounded by a high fitness neighbourhood and probably it will not be selected. In order to prove this statement, a statistical search is carried out in the neighbourhood of the optimal solution, choosing random errors for each parameter. The error for the ith parameter is randomly set with a normal distribution around the optimum, considering suitable standard deviations σ_i, such errors are chosen in order to have more than 90% of gears within the tolerance for the single parameter. Here tolerances are 1 μm for magnitudes and 0.2° for roll angles. The reliability analysis is carried out on a population of 100 pinion-gear pairs with random errors. Starting from the optimal configuration "D", which has a STE peak to peak equal to 1.63 μm, the result of the analysis is a mean value equal to 2.1 μm and a standard deviation equal to 0.29 μm; none of the 100 configurations has a peak to peak smaller than the optimum found with GAs, this confirms the good performance of the algorithm.

Assuming that the probability distribution of the peak to peak increase is Gaussian (the perturbation is given in the neighbourhood of the optimum), the probability of having such increase within a certain range is estimated; results are reported in Table 6. As expected from GAs, the optimum is reliable; indeed, 6.5% of samples do not give any appreciable variation of the objective function, almost the 80% of samples give a

Table 6
Reliability analysis

Maximum peak to peak increase (%)	Maximum peak to peak (μm)	Probability (%)	Peak to peak reduction (%)
< 0	1.6323	6.5	> 86.0
0–20	1.9588	28.2	86.0–83.2
20–40	2.2852	41.8	83.2–80.4
40–60	2.6117	20.2	80.4–77.6
60–80	2.9381	3.1	77.6–74.8
80–100	3.3265	0.1	74.8–71.4

reduction of performances less than 60%. It is to note that an increasing of peak to peak of 60% is not a bad result; indeed, such increasing with respect to the optimum is still a 70% less than the non optimized case.

7. Conclusions

In the present study, an optimization approach based on Genetic Algorithms is proposed to improve gear dynamic performances toward noise reduction. Linear and parabolic profile modifications are considered and compared by means of several optimization strategies based on static nonlinear FEM analysis.

Genetic algorithms are proven to be an effective optimization tool to design reliable profile modifications for reducing the gears vibration, i.e. they allow a strong reduction of the vibration amplitude over a wide frequency range, as proven by dynamic analyses.

The reliability analysis shows that optima found by the present GA are robust in terms of uncertainties in the manufacturing parameters.

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